

ASTRONOMY, ASTROLOGY, OBSERVATORIES AND CALENDARS

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The Persian and Indian origins of Islamic astronomy

The history of astrology in the countries of Central Asia goes back to ancient times. Excavations at the Koy-Krylgan-kala site (fourth century B.C.–fourth century A.D.) show that this religious structure was designed in such a way that it could, like the Babylonian ziggurats, be used for astronomical observations. Archaeological finds at the site included fragments of clay discs and flattened rings, a combination which recalls the reconstruction of a Greek astrolabe with a circular alidade made by O. Schirmer according to the description by al-Bīrūnī.¹

We know that the Khwarazmians had their own era and their own calendar, which was distinguished by its great precision. Keeping such a calendar necessitated stationary observations of the daily movements of the moon and sun and the positions of the stars at the vernal and autumnal equinoxes and the summer and winter solstices, which in turn presupposed a thorough knowledge of astronomy. Al-Bīrūnī provides some information about astronomy in ancient Khwarazm. He says that an astronomer ‘was called *akhtarvenik* in

¹ Schirmer, 1926–7, pp. 43–6, 63–79.

the language of the Khwarazmians’ and that ‘the Khwarazmians knew the constellations better than the pre-Islamic Arabs’.² In the same passage he complains bitterly that ‘the learned Khwarazmians who used lunar stops and knew very well how to observe and draw conclusions from them have died out’.

Somewhat more extensive information has been preserved about astronomy in pre-Islamic Iran than in pre-Islamic Khwarazm. We know of the existence of a Pahlavi translation of certain writings of Greek origin. According to this information, one of the early Arabic translations of Ptolemy’s *Almagest* (first–second century) was made by ʿAlī b. Rabbān al-Tabarī from the Pahlavi. Some Indian influence also appears to have been noticeable in pre-Islamic Iran. This can be seen from the example of the Sasanian astronomical treatise *Zīj al-Shāh* or *Zīj-i Shahriyār* [Astronomical Tables of Shahriyār] (from the Pahlavi *Zīg-i Shahriyār*). The first version of this *Zīj* is thought to date back to about 450 and to have been dedicated to the Sasanian emperor Yazdgird II (438–57).³ Later, in the year 556, under the reign of Khusraw I Anūshirvān (531–79), ‘it was re-edited and expanded by the addition of information of Indian and Hellenistic origin’.⁴ Its final version was made under Yazdgird III (632–51), when it was given the name of *Zīg-i Shahriyār*, or Shahriyār’s *Zīj*. E. S. Kennedy has established that this Sasanian *Zīj* includes parameters borrowed from the Indian *siddhantas*, in particular from the *Surya-siddhānta* and the *Brahma-sphuta-siddhānta* of Brahmagupta.⁵

The *Zīj al-Shāb* in its complete form has not come down to us. Separate fragments of this *Zīj* and references to it are to be found in the works of al-Bīrūnī.⁶ On the basis of these, it may be concluded that the original included a large number of tables of Indian and, possibly, Babylonian origin. The compilers of this work understood the term *zīg* to mean a composition of a tabular nature because in Pahlavi this word originally meant ‘cellular’ or ‘reticular’. Later, this word in its Arabized form *zīj* became synonymous with the Greek *kanōn* (canon) and came to signify an astronomical work containing tables.

The sources mention another work in the Pahlavi language which also has not come down to us. This is the *Bizidaj*, a text of an astronomical nature. Al-Bīrūnī drew upon it.⁷ It has been established that the *Bizidaj* is the translation into Pahlavi of the *Anthology* of Vettius Valens (second century A.D.).⁸ The astronomical treatise *The Eighty-Five Chapters*,

² Al-Bīrūnī, 1957.

³ Pingree, 1963, pp. 229–49.

⁴ Pingree, 1973, p. 34.

⁵ Kennedy, 1958, p. 246.

⁶ Al-Bīrūnī, 1959, pp. 28–9, 58–9.

⁷ Al-Bīrūnī, 1995, pp. 4–5.

⁸ Nallino, 1944, pp. 238–40.

ascribed to the mythical ancient Egyptian physician and astrologer Hermes Trismegistus, was also known in Sasanian Iran.

Indian astronomy may have had some influence on the development of astronomy in the caliphate. A large group of eminent astronomers was active in India in the fourth to the seventh century, but details of a possible transmission of their works are lacking.

The earliest *zīj*s in Arabic appeared in the eastern part of the caliphate, i.e. in the southern region of Central Asia. These works had either been translated into Arabic or had been written in Arabic on the basis of ancient local scientific tradition. Thus al-Bīrūnī mentions an old *zīj* seen by him at the house of ʿAlī b. Muhammad al-Washjirdī at Ghazna. This *zīj* was constructed according to the era of Diocletian, which began in 284, and contained references to astronomical observations made between 709 and 719, these observations being entered at the end of the *zīj* not by the author himself but by one of the book’s owners. The *zīj* must therefore have been written at a considerably earlier date, i.e. in the seventh century. In the second half of the eighth century, under the caliph al-Mansūr (754–75), the city of al-Mansura which the Arabs founded in western India became a centre of scholarship. The *Zīj al-Harkand* was written down in Arabic; according to al-Bīrūnī, this was a translation into Arabic of the *Khandakhādyaka* of Brahmagupta (598–665).⁹ Evidently the translation was not entirely accurate, as the planetary equations had been garbled, and al-Bīrūnī therefore made a new translation of the work into Arabic.¹⁰ Two *zīj*s compiled at Kandahar are also known. They are the *Zīj al-Jāmiʿ* and the *Zīj al-Hazūr*. These, too, are thought to be derived from the *Khandakhādyaka* of Brahmagupta. Al-Bīrūnī also mentions the *Zīj al-Harkand*, in which the calculations are made for the year 110 of the ‘era of Yazdgird’, i.e. 742.¹¹

The true flowering of scientific and translating activities took place in Baghdad, the centre of the caliphate, under the same caliph, al-Mansūr. According to the biobibliographer Ibn al-Qiftī (twelfth-thirteenth century), a mission from India, whose members included the scholar Kanka, arrived in Baghdad in the year 156 of the Hijra (A.D. 773) (according to al-Bīrūnī, in 154/771). Kanka carried with him certain Indian books, among them the *Brahma-sphuta-siddhānta* of Brahmagupta. Ibrāhīm al-Fazārī translated it into Arabic by order of the caliph, and this translation later became known as the *Great Sindhind*. The translator-scholars Yaʿqūb b. Tāriq and Māshʿallāh, also active in Baghdad at the time of al-Mansūr, produced translations and editions of Indian astronomical treatises,

⁹ Al-Bīrūnī, 1959, pp. 37–8, 152.

¹⁰ Boilot, 1955, pp. 203–4.

¹¹ Kennedy, 1956, p. 137; Pingree, 1970, pp. 103–23.

Ibn Tāriq being responsible for an edition of Brahmagupta's *Khandakhādya* and the *Zīj al-Harkand* mentioned above.¹²

Jābir b. Hayyān (721–89) was the first among the Arabic-speaking scholars of the time of al-Mansūr to turn his attention to the works of Greek scholars. He translated works by Euclid, Ptolemy, Theo of Alexandria, Alexander of Aphrodisia and others. One of his major astronomical works was a commentary on Ptolemy's *Almagest*.¹³ During the time of the caliph Hārūn al-Rashīd (786–809), the *Almagest* was translated into Arabic twice. However, all these translations and editions were made via the intermediary of the Syriac.

In the ninth century, under al-Ma'mūn (813–33) and his successors, several translations of the *Almagest* directly from the Greek were made by the translators Hajjāj b. Matar, Ibrāhīm b. al-Sāmī, Husayn b. Is'hāq, °Umar b. Farrukhān, Thābit b. Qurra, al-Nayrīzī and al-Battānī. Al-Ma'mūn became caliph in Merv in 813 and ruled the caliphate from that city until 819. Here he gathered around him astronomers from Ferghana, Chach, Khwarazm, Khurasan and what is now Afghanistan. In 819 he moved to Baghdad and was followed there by Yahyā b. Abī Mansūr, Khālid b. °Abd al-Malik al-Marwarrūdhī, °Abbās b. Sa°id al-Jawharī, Abū Tayyib Sanad b. °Alī, Muhammad b. Mūsā al-Khwārazmī, Ahmad b. Muhammad b. Kathīr al-Farghānī, Ahmad b. °Abd Allāh Habash al-Hasib, °Abd al-Hamīd Ibn Turk al-Khuttalī and others.

All these scholars were astronomers. They conducted stationary observations at observatories in the Shammasiyya district of Baghdad and at the Dayr Murrān on Mount Qasyun near Damascus. They also measured the length of one degree of the earth's meridian in the Sinjar desert of northern Syria. The best known among them are al-Khwārazmī and al-Farghānī. The former compiled his *Zīj* on the basis of al-Fazārī's *Great Sindbind* and of Theo of Alexandria's fourth-century adaptation of the *Almagest*. This work has been the object of many commentaries by Muslim scholars. Around the year 1000 it was edited by the scholar of Muslim Spain, Maslama al-Majrītī, and this was twice translated into Latin in Spain in the twelfth century by Adelard of Bath and by Petrus Alfonsus; such translations later made their way across medieval Europe and influenced the work of Renaissance scholars.

Al-Farghānī (d. 860) was the author around the year 840 of a *Kitāb Usūl °ilm al-nujūm* [Book on the Elements of the Science of astronomy] (and variant titles), which was a popularized version of Ptolemy's *Almagest*. The work was twice translated into Latin in the twelfth century: first in 1145 by John of Seville and again in 1175 by Gerard of Cremona.

¹² Nallino, 1944, p. 204.

¹³ Sezgin, 1978, pp. 129–34.

These translations were printed in Europe from 1493 onwards and al-Farghānī's work was for a long time Europe's principal textbook of astronomy.

Al-Ma'mūn's Bayt al-Hikma (House of Wisdom) functioned until the end of the tenth century. In 1900 the German historian of science, H. Suter, published a biobibliographical work in which he cited the names of some 500 astronomers working there in the ninth and tenth centuries; J. Ruska commented that the scholars listed 'originated almost without exception in Khurasan, Transoxania, Bactria and Ferghana'.¹⁴

Genuinely outstanding results in astronomy were achieved by the great Khwarazmian scholar al-Bīrūnī (973–1048). Over 80 of his more than 150 works are devoted to astronomy and related subjects.¹⁵ In his astronomical writings, al-Bīrūnī sums up three centuries of development of astronomy in the Arab caliphate. He submits all earlier translations of the *Almagest* and of the Indian astronomical texts to critical review and evaluation. In his encyclopedic *Tahqīq mā li 'l-Hind* [Inquiry into What is to be Found in India], he gives full information about all Indian astronomical writings, their Arabic translations and the Indians' achievements in the sphere of astronomy. In his *al-Qānūn al-Mas'ūdī* [Canon of (Sultan) Mas'ūd], composed of 11 books, al-Bīrūnī assesses the achievements of Arabic-speaking astronomers. In this work he divides Muslim astronomers into two groups: adherents of the Hellenistic tradition and those of the Indian tradition. While holding the Indians' achievements in astronomy in high esteem, he points out the erroneous nature of their views in such matters as the theory of planetary motion, the distances from the earth to the planets and the dimensions of the earth. For this reason, he does not agree with the adherents of the Indian tradition and counts himself among those of the Hellenistic tradition. Unfortunately, al-Bīrūnī's more substantial astronomical works have not been preserved. However, their titles give grounds for supposing that they too were devoted to questions of Hellenistic and Indian astronomy.

During the time of Timur and his dynasty, Samarkand became a major centre of scholarship not only for mathematics (see Chapter 6, above) but also for astronomy, especially under Ulugh Beg (1394–1449). Besides the ruler himself, this school included the leading astronomers of the fifteenth century: Qādīzāda Rūmī (d. 1430), Jamshīd al-Kāshī (d. 1429) and °Alī al-Qūshjī (d. 1475). The first major astronomical work produced by this school was the *Zīj-i khāqānī*, a completion and emendation of the Il Khanid *zīj* (*Zīj-i khāqānī dar takmīl-i zīj-i Ilkhānī*) written by al-Kāshī for Ulugh Beg's library in 1417. The most important astronomical work of the school of Samarkand was, however, the *Zīj* of Ulugh Beg, on which he began working as early as 1414. In the initial stages of the work, he was assisted

¹⁴ Ruska, 1927, p. 127.

¹⁵ Matvievskaia and Rozenfeld, 1983, pp. 264–95.

by Qādīzāda Rūmī and Jamshīd al-Kāshī. After their death the work was completed in 1444 with the assistance of °Alī al-Qūshjī. The whole of medieval astrology is summed up in this work and all its tables are compiled with great precision. The *Zīj* has been the subject of many commentaries by Muslim scholars and came to the notice of European scholars as early as the sixteenth century. All tables in it were compiled for the latitude of Samarkand as computed by Ulugh Beg (equal to $39^{\circ}37'23''$) (and for the first of Muharram of the year 841 of the Hijra, which corresponds to 5 July 1437 A.D.

Astrology

In Central Asia, as in other Muslim lands, astrology existed as a branch of science side by side with astronomy. Although it is stated in Qur'an 27:65 that 'No one but God shall know the future' interest in astrology was always intense, and it was almost universal for Islamic rulers to govern their decisions and actions by astrological considerations.

Two trends in astrology may be distinguished. The first is based on measurements and mathematical theory, i.e. mathematical astronomy, while the second is magical and irrational, unsupported by mathematical calculations of any kind.¹⁶ Many Islamic scholars were critical of astrology and its practitioners, evidently having in mind those of the second kind. Thus al-Bīrūnī first of all draws a clear distinction between astronomy and astrology. He describes astronomy as *°ilm al-nujūm* (science of the stars) or *°ilm hay'at al-nujūm* (science of the structure of the stars), the word *°ilm* emphasizing that astronomy is a science. As for astrology, he describes it by the terms *sinā° at al-nujūm*, *sinā° at ahkām* and *sinā° at ahkām al-nujūm* (the art of star-counting, the art of divination and the art of predicting the future by the stars); in other words, astrology is an art or practice, distinct from the science of astronomy. Furthermore, in all these terms the word 'art' carries implications of 'swindle', 'machination', and so on. For these reasons, al-Bīrūnī criticizes astrology in many of his works, going so far as to devote to that purpose a special treatise entitled, 'Warning against the Art of Fraudulent Divination by the Stars'. He openly condemns astrological predictions in his *Tahdīd nihāyāt al-amākin*, or *Geodesy*: 'Generally speaking, the art of prediction has weak foundations, and the theses derived therefrom are likewise weak. The measurements taken within it are confused, and suppositions prevail over reliable knowledge.'¹⁷ A harsher criticism of astrology and astrologers is to be found in Book II of his *al-Qānūn al-Mas°ūdī* devoted to astronomy:

¹⁶ Sayili, 1960, p. 47.

¹⁷ Al-Bīrūnī, 1966, p. 260.

The art of interpreting the verdicts of the stars, to which this book is confined, is essentially autonomous in that it carries its 'value' exclusively within itself. As a rule, it attracts only the hearts of those who imagine pleasure to consist in being free from physical suffering and profit to lie only in worldly goods. If you do not seek those goods you will be repelled by this art and its predictions, its rules and its practitioners.¹⁸

Despite such criticisms, astrology nevertheless remained very popular in the countries of Islam in pre-modern times.

In his *Kitāb al-Taḥḥīm li-awā'il sinā'at al-tanjīm* [Book of Instruction in the Elements of the Art of Astrology],¹⁹ al-Bīrūnī further criticizes astrology. He discusses in detail the astrological doctrines of the Greeks, India, Iran and Central Asia and the astrological significance of various signs of the Zodiac, heavenly bodies and astrological 'houses'. Emphasizing the fact that the same astrological question is treated differently, and sometimes even in contradictory fashion, by various astrologers in these countries, al-Bīrūnī exposes the falsity of astrology as a science. Where possible, he reverts to mathematics and astronomy. In particular, he describes in detail the 'module-based' arithmetical operations needed for the calculation of astrological horoscopes. All obscurities, such as 'stars which have an adverse influence on vision', are listed and methods of measuring time with the help of clepsydras, or water-clocks, are set out in detail.

With the exception of the obvious effects of the sun's rays, al-Bīrūnī does not believe claims that events on earth are influenced by heavenly bodies. If astrological predictions are, in principle, an algorithm which makes it possible to determine by the position of the planets at a given moment the fate of a person born at that moment, or to determine whether that moment is favourable for some particular undertaking, then (since the direct application of this algorithm may lead to conclusions completely unrelated to reality) the astrologers introduce an element of arbitrariness, for instance through the *khaylaj*, which is chosen as one of five possible points on the ecliptic; they then carry out a number of astrological calculations which allow them to make the astrological prediction fit the previously known result. The question of how the result of a prediction can be known in advance is dealt with in the book's last section, where al-Bīrūnī advises readers to pay careful heed to all instructions and actions of the questioner and, in turn, to ask him additional questions, elucidating all the circumstances and, in substance, arriving at the desired conclusion without any observations or calculations. The same conclusion can then be reached with the help of the astrological algorithm.

¹⁸ Al-Bīrūnī, 1976, p. 449.

¹⁹ Al-Bīrūnī, 1934.

In effect, astrology operates with the same concepts as astronomy but endows them with astrological significance. For example, the word ‘horoscope’ (from the Greek *horos* ‘time’ and *skopein* ‘to determine’) initially meant the ascent or ascending degree of the ecliptic. This was used in determining the time of a given event, the spherical co-ordinates of the planets and other important points of the celestial sphere. Later, the term acquired a purely astrological significance. That is why certain medieval scholars devoted parts of their astronomical writings to the subject of astrology. In one of the earliest astronomical texts, the *Zīj* of Muhammad b. Mūsā al-Khwārazmī (*fl.* first half of ninth century), Chapters 32, 36 and 37 are devoted to astrology.²⁰ The last of the four books of the *Zīj* of Ulugh Beg, the greatest of the later scholars of Central Asia, is also given over to astrology.²¹

Observatories and astronomical instruments

The first experiments with observation instruments date back to the time of al-Ma’mūn in Baghdad and Damascus (see above). No detailed information has, however, been preserved on these except for fragmentary mentions by al-Bīrūnī in his *Qānūn* and his *Geodesy*.

We have more details about the activities of the astronomer Abū Mahmūd Hāmid al-Khujandī (d. c. 1000), who worked at Rayy and was the author of, *inter alia*, a pioneer work on the astrolabe, *Risāla fī ‘Amal al-āla al-shāmila* [Treatise on the Construction and Utilization of the Astrolabe]. Al-Khujandī is believed to have proved the sinus theorem for the spherical triangle,²² and became famous as the inventor of the sextant, which he called *suds-i fakhrī* (Fakhrī’s sextant) in honour of his patron Fakhr al-Dawla, the Buyid ruler of Rayy. Prior to the invention of this sextant by al-Khujandī, the astronomical instruments used for measuring the geographic latitude of the place of observation, the angle of obliquity of the ecliptic to the celestial equator, etc., were based on visual observation of the meridian altitudes of the heavenly bodies. Al-Khujandī’s sextant was, however, based upon an entirely new principle. The star (the sun) is not observed by the observer’s eye; its rays pass through a dioptric lens placed in the upper part of a darkened chamber and the sun itself is ‘caught’ as a reflection on the scale of the sextant’s arc, which is located in the lower part of the chamber. The arc must be placed strictly within the meridian plane, while the dioptric lens is placed at the centre of an imaginary circumference of which the graduated arc represents one-sixth. The scale of the arc of ‘Fakhrī’s sextant’ was graduated at intervals of 10 seconds.

Here is how al-Bīrūnī describes al-Khujandī’s sextant in his *Geodesy*:

²⁰ Al-Khwārazmī, 1983, pp. 49, 54, 55.

²¹ Ulughbek, 1994, pp. 255–77.

²² Matvievskaya, 1972, pp. 85–6.

By order of Fakhr ad-Dawla, Abū Mahmūd Hāmid b. al-Khidr al-Khujandī built on Mount Tabarak in the outskirts of Rayy two parallel walls along the meridian, the distance between the walls being seven cubits (approx. 3.5 m). Between the walls he constructed an arch with a dioptric lens on top of it, the aperture of the lens having a diameter of one span (0.3 m). He made the centre of this lens the centre of the sixth part of a circumference likewise placed along the meridian between these two walls and having a diameter of eighty cubits (approx. 40 m). He paved this sextant with wooden boards, faced it with copper and marked each degree of the circumference with three hundred and sixty equal segments, each of which equals ten seconds. The sunlight passed through this dioptric lens and fell upon the meridian line. Abū Mahmūd made a ring of the dimensions of the circumference of a sunbeam falling upon the ground, having established its centre by two intersecting diameters. He placed the circumference of the ring upon the circumference of the reflection of the sun and, using the centre of the ring, determined the distance between the sun and the zenith.²³

In his treatise, the *Risāla fī Tas'hīh al-mayl wa-^card al-balad* [On the Precise Determination of the Maximum Declination and latitude of a Town], al-Khujandī states that, when observing the meridian altitudes of the sun on the days of the summer and winter solstices of the year 994, he found the latitude of the town of Rayy to equal $35^{\circ}34'15''$ (with an error of less than $5'$) and the obliquity of the ecliptic towards the equator to equal $23^{\circ}32'32''$ (with an error of $1'51''$).²⁴ Subsequent to this, a number of astronomical instruments were constructed by al-Bīrūnī, some of which are described in his *Geodesy*;²⁵ he also wrote several treatises describing astrolabes and other astronomical instruments.²⁶

It was Ulugh Beg who had the honour of further improving Fakhrī's sextant. In 1420 he undertook the construction of an observatory on Mount Kuhak at Samarkand, which was completed in 1429. The observatory's main instrument was a meridional arc constructed on the principle of Fakhrī's sextant. But it was not a simple copy of al-Khujandī's instrument. In the first place, it differed from that instrument by its dimensions. Its diameter was twice that of Fakhrī's sextant and equalled 84 m.²⁷ The arc of Ulugh Beg's sextant was a quadrant. The working part of the instrument was graduated from 20° to 80° . It is thought that the main instrument of Ulugh Beg's observatory was used as a quadrant as well.²⁸ By means of this instrument, the latitude of Samarkand was found by him to equal $39^{\circ}37'23''$ and the angle of obliquity of the ecliptic to the celestial equator to be $23^{\circ}30'$

²³ Al-Bīrūnī, 1966, p. 133; Bulgakov, 1972, p. 48.

²⁴ Bulgakov, 1972, p. 49.

²⁵ Al-Bīrūnī, 1966, pp. 111, 112, 113, 151.

²⁶ Bulgakov, 1972, pp. 420–2.

²⁷ Kori-Nieziy, 1971, p. 97.

²⁸ Dzhahalalov, 1950, p. 81.

'17". Besides Fakhrī's sextant, instruments used at Ulugh Beg's observatory included an armillary sphere, a sinus quadrant, dioptric lens instruments and azimuth quadrants.²⁹

Calendars

In the countries of Central Asia, as in other Islamic lands, the most widely used calendar was the lunar Hijri one. This calendar is based on a year composed of 12 lunar months. The length of a lunar month is taken to be the period between two new moons, which equals 29.5306 days. The duration of the year according to the Hijri calendar is just over 354 days. The fractions by which this number is exceeded are added together to form an extra day which is intercalated every second or third year, this year being considered as a leap year (*kabīsa*).

In the countries of Central Asia, the Persian, Sogdian and Khwarazmian calendars were known from ancient times. All these were solar calendars in which the year was composed of 12 months of 30 days. Five days were added on at the end of the year or after the tenth month, so that the year had 365 days. The first day of the year was taken to be 21 March, the day of the vernal equinox (*Nawrūz*). This day marked the beginning of the first month of the year – Farvardin for the Persians, Navsard for the Sogdians and Navsarju for the Khwarazmians.³⁰ The Khwarazmian and Sogdian calendars were abandoned following the definitive establishment of Islam in the region.

The Persian era began on 16 June 632, the beginning of the year of the accession to the throne of the last Sasanian emperor, Yazdgird III (632–51), hence the era was also known as that of Yazdgird. According to the Persian solar calendar, however, the year began on 21 March. This calendar had no leap years and for that reason had eventually to be reformed. The reform was carried out in 1079 by order of the Seljuq sultan Jalāl al-Dīn Malik Shāh (1072–92) by a group of astronomers headed by ʿUmar Khayyām: the new calendar system was called the 'Malikī era' or the 'Jalālī era' in honour of the sultan.³¹ This calendar was in use in Iran until the middle of the nineteenth century.

²⁹ Yusupova, 1979, pp. 53–7.

³⁰ Al-Bīrūnī, 1976, pp. 223–60.

³¹ ʿUmar Khayyām, 1961, pp. 19–35.